Designing Problem Sets and Exams

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In this session we will discuss aspects of writing new problems:

• Principles for designing effective problem sets, exams, and in-class worksheets
• Practical issues: making problems easy to grade; homework vs. exams
• Apply these ideas by analyzing examples of effective problem sets

I. Contexts where you might write new problems:

• Problem sets/homework – Tends to have relatively narrow focus (1-2 weeks of material), assumes students may not yet understand material
• Exams – Tends to have broad focus (1-3 months of material), assumes students already understand material
• In-class worksheets (“lecture tutorials”) – You can prepare a worksheet and have students work together on the problems during recitation. This promotes active learning and helps students assess their own learning.

II. Different types of scientific problems:

• Multiple choice, solving equations, plotting and interpreting data, word problems, short answer questions, essays, design problems, proofs, computational/coding problems, ...

III. Things to know before designing a problem set/an exam:

• How important is: Critical thinking? Rote memorization?
• Is it important that students integrate across concepts?
• Is it important that students know details?

IV. Useful principles for all contexts:

1. Top-down design (start by outlining your goals for the problem set or exam)
2. Conceptual scaffolding (build from simple to complex)
3. Bloom’s taxonomy (write problems that use different levels of understanding)
4. Breadth, depth, and definition (“Scientific Problem Space”)

Top-down design:

Before writing any problems, start by writing down your goals for the problem set or exam. Doing this ensures that you cover all the important topics and skills in a balanced way. (And avoids a final exam where 80% of the exam covers 20% of the course.)

Example goals for a calculus I problem set (covering 1 week of material):

• Practice skills: integrating trigonometry functions, using trigonometric identities
• Synthesize with previous learning: integrate equations with combinations of trig functions and polynomials
• Apply to real-world example: cycloid (motion of edge of wheel)

Example goals for a calculus I final exam (covering 1 term of material):
• Calculate derivatives and integrals of various types of single-variable functions
• Translate word problems (distance optimization, velocities/accelerations) into calculus problems
• Proofs: trigonometric identities, epsilon-delta proofs
• Explain meaning of fundamental calculus theorems in words & graphically

Conceptual scaffolding:
Build from simple to complex problems. This helps students learn to think like experts.

Research on STEM education shows that experts tend to solve problems by first identifying the relevant concepts, whereas students tend to look for equations that they can plug numbers into or they look for similar example problems that they can imitate.

Conceptual scaffolding: have students practice relevant concepts on simple problems first, then give them a more complex problem that requires the concepts from the simple problems.

Bloom’s taxonomy:
Give problems at different levels of difficulty that use different types of understanding. “Bloom’s taxonomy” is a way of describing different types of understanding. Creating problems that test a few different levels from Bloom’s taxonomy is a good way to create a well-rounded problem set or exam. Basic and complex levels are both important!

Some types of problems at different levels:
**Evaluation/creation**: apply class principles to research; make judgments between ideas; mathematical proofs; design a circuit or algorithm

**Synthesis**: combine ideas from multiple weeks in class or multiple subjects

**Analysis**: make inferences, provide evidence; draw conclusions from a plot of research data

**Application**: apply knowledge to new situation, solve a word problem

**Comprehension**: describe in own words, short answer questions

**Knowledge**: practice using an equation, make a list of facts, classify items
Breadth, depth, and definition (“Scientific Problem Space”):

X = Depth: how many details does it require?
Y = Breadth: how many concepts/topics does it span?
Z = Definition: how much information/constraints are provided?

Try to determine where in problem space you want to be and design the problem to fit those requirements.

Tips for using scientific problem space:
• Use problems with a range of breadth and depth. Even the lowest students will not be overwhelmed and will have somewhere to start while the best students will be challenged.
• Use both qualitative and quantitative problems.
• Well-defined problems are easier to grade (but sometimes less defined problems are good to challenge students or give them a chance to be creative).